ImAppNIO STSM Report

Research visit between Vilnius University (Lithuania) and University of Almeria (Spain)

STSM Applicant: Dr. Ernestas Filatovas, Vilnius University, Institute of Mathematics and Informatics, Vilnius, Lithuania, ernestas.filatovas@mif.vu.lt

STSM topic: Improvement of preference-based evolutionary approaches for multi-objective optimization.

Host: prof. Dr. Gracia Ester Martín Garzón, University of Almeria, Almeria, Spain, gmartin@ual.es

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Dr. Ernestas Filatovas (Vilnius University) visited the research group "High-Performance Computing – Algorithms" (Almeria university) from January 16th to February 1st. The goal of the STSM visit was to improve theoretical basis and applicability for practical problems of the preference-based evolutionary algorithm for solving multi-objective optimization problems. Moreover, one of the main objectives of this STSM was to strengthen the collaboration between Institute of Mathematics and Informatics of Vilnius University and the research group "High-Performance Computing – Algorithms" from University of Almeria.

During the STSM visit, I have spent two and a half weeks under the supervision of professor Gracia Ester Martín Garzón. The main research lines of this group are High-Performance Computing, Global, and Multi-objective optimization, Systems Modelling and Resolution, Energy Efficient Computing.

Many real-world problems are multi-objective, where several conflicting objective functions must be minimized. Usually, there is no solution which would be the best for all objectives, however, a set of optimal solutions in a multi-objective sense exists. These solutions are defined as the solutions where none of the objective values can be improved without deteriorating other(s). Such a set of solutions is called the Pareto set and the corresponding set of objective vectors, the Pareto front. Determination of the Pareto front is the main goal of multi-objective optimization, however, often it is impossible to identify the exact Pareto front due to the reasons as continuity of the front or complexity of the problem being solved. Therefore, algorithms that approximate the Pareto front are widely-used. Evolutionary Multi-objective Optimization (EMO) approaches are commonly employed for this task [1, 2].

The set of obtained solutions approximating the entire Pareto front is presented to the Decision Maker (DM). However, EMO algorithms are computationally expensive and time-consuming. Additionally, only a reasonable number of solutions should be given to the DM so that he/she can make an adequate decision avoiding the usually complex analysis of a large amount of information and reducing cognitive burden. Moreover, the DM is commonly interested only in a certain part of the Pareto front and prefers to explore that part deeper. Thus, incorporation of DM's preferences into EMO algorithms has become a relevant trend during the last decade [3-7].

The DM's preference information is usually expressed as a Reference Point (RP), as it is the simplest way to specify the preferences. Therefore, a preference-based EMO algorithm, during its execution, emphasizes solutions close to the RP. In particular, the region of interest (ROI) is a part of the Pareto front determined by the RP provided by the DM (see Fig. 1). During exploration process, the DM provides several different RPs, therefore, it is important to ensure that the obtained solutions by a preference-based EMO algorithm are in the ranges, defined by the RP and meanwhile cover the whole ROI. However, only a few preference-based EMO approaches are able to obtain well-distributed solutions covering the complete ROI – RD-NSGA-II [4], WASF-GA [7] algorithms. Other popular developed algorithms as R-NSGA-II [3], r-NSGA-II [6], etc. require tuning of the parameters to obtain solutions only in the ROI and to cover it well. These parameters must be set for each problem and for each reference point, independently, that is complicated when solving real-world MOO problems. RD-NSGA-II, WASF-GA approaches use values of an achievement scalarizing function (ASF) to classify the individuals at each generation into several fronts. However, only WASFGA is able to cover the whole ROI for problems with 3 or more objectives. This algorithm currently is the best known for approximation the region of interest of the Pareto front. Though, it requires minimizing

ASF on each iteration. Moreover, our primary experimental investigation [1] has shown that WASFGA demands rather many iterations to get good enough approximation. This disadvantage is crucial when solving optimization problems with a high computational burden.

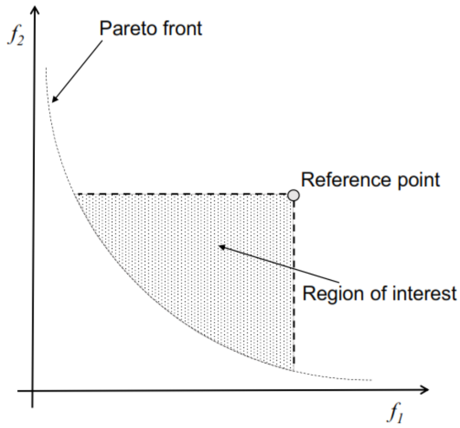

Fig. 1 Region of interest

Recently, together with research group “High-Performance Computing – Algorithms”, we have developed an initial version of the preference-based EMO algorithm that considers the DM’s preference information expressed by means of RPs and approximates ROI [8]. The algorithm was experimentally investigated on several test problems. The obtained results were promising and showed potential for the further development and application of the algorithm. During the STSM visitwe focused on the improvement of the algorithm, experimental investigation as well as its application.

At the beginning of my mission together with prof. dr. Gracia Ester Martín Garzón, Dr. Juana López Redondo, Dr. Gloria Ortega, and Dr. José Fernández Hernández we have discussed the currents state of our common researches, as well as, lines of our further collaboration, and also composed a detailed work plan of the pre-experimental and experimental investigations to be carry out during and after the STSM.

We started from improvement and extension of the theoretical background of our developed algorithm called CHIM-NSGA. The algorithm combines ideas of the well-known EMO algorithm NSGA-II [9] and the NBI method [10]. NBI is based on the concept of Convex Hull of Individual Minima (CHIM) – the set of all convex combinations of the individual global minima of the objective functions. In a two-objective case, the CHIM is a line segment (see Fig. 2). The NBI method combines all the objective functions into a single objective one via a weighting vector and finds optimal solutions by minimizing the single objective for various values of weighting vector. As a result, NBI method finds a uniformly spread set of Pareto optimal objective vectors. Thus, we incorporated CHIM-based selection into a preference-based EMO algorithm in order to achieve a good distribution of the obtained solutions in the ROI.

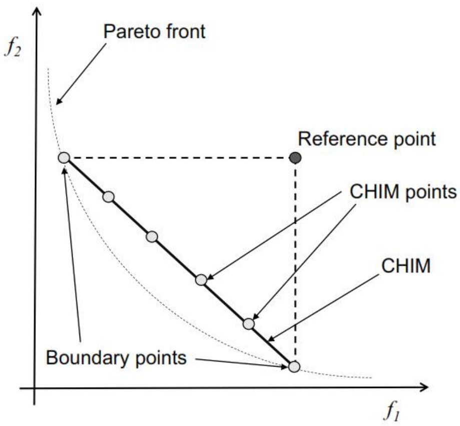

Fig. 2 The CHIM of the region of interest

As we deal with a preference-based approach, before running the proposed CHIM-NSGA algorithm, the DM's preferences should be provided by means of an RP. Then the boundary points (see Fig. 2) of the region of interest are obtained by a suitable single-objective optimization algorithm. This ensures that the appropriate region of interest is approximated.

The steps of the CHIM-NSGA algorithm are as follows:

1. A random initial population P_0 consisting of N decision vectors is randomly generated.
2. At iteration t, a new offspring population Q is created by applying genetic operators (crossover and mutation) to the individuals of the parent population.
3. The parent and offspring populations are combined into one joint population P_t.
4. The new population is sorted into different non-domination levels (so-called fronts) by a non-dominated sorting procedure (as in NSGA-II algorithm).
5. The obtained joint population is reduced to the size N of the parent population by leaving the individuals from the best non-domination levels. If not all individuals from the last level can be selected for the next generation, then CHIM line points are generated and the individuals closest to those points are selected. The CHIM line points are evenly distributed between the boundary points (see Fig. 2). The number of points on the CHIM line is equal to the number of points to be selected for the new population from the last non-dominated level that cannot be completely selected.
6. If the termination condition is not satisfied, then the process is repeated from Step 2 considering the reduced population as the parent population in the next iteration.

The next stage of STSM visit was the experimental investigation. The performance of the CHIM-NSGA algorithm is experimentally researched and compared to the state-of-the-art WASF-GA and R-NSGA-II algorithms. A set of well-known test problems has been considered: it includes the bi-objective problems ZDT1-ZDT4, ZDT6, and the tri-objective problems DTLZ1-DTLZ6 [11]. Each experiment has been performed for 30 independent runs using different initial populations and average results have been evaluated. We selected a population size of 100 individuals and 200 generations for the problems with two objectives, and a population size of 200 individuals and 400 generations for the three-objective problems, as such population sizes are enough to sufficiently represent an approximation of a region of the Pareto front. We selected a relatively small number of generations in order to find a good enough approximation in reasonable computational time. The performance of the PMOEAs has been evaluated using the following metrics: Generational Distance (GD) [1] for estimating convergence to the true Pareto front, Spread [1] for estimating the distribution evenness of the solutions. PR metric [12] is also considered – it evaluates the percentage of solutions that lie into the DM's region of interest. Preference information provided by the DM that is expressed as an RP is required for the evaluated algorithms. The used RPs and the number of objectives and variables for each test problem considered are presented in Table 1. The mean values of PR for all performed runs with each test problem are presented in Table 2. It can be seen in Table 3 that all the solutions obtained by CHIM-NSGA enter into the ROI for all the test problems except DTLZ3 (the most complex one). This means that the fixed sizes of populations and generations are enough to find a crowded set of solutions when running the algorithm. Thus, WASF-GA is not able to obtain all the solutions into the ROI within the fixed number of generations.

The mean values of Generational Distance and Spread metrics and their confidence intervals (95% confidence level) for each test problem are presented in Tables 3-4, respectively. The best (lowest) average values of the calculated metrics are marked in bold. According to the GD metric, the proposed CHIM-NSGA algorithm approximates the ROI better than the R-NSGA-II and WASF-GA algorithms for all the analyzed problems except in the ZDT6 and DTLZ3 cases (see Table 3). Notice, however, that in those cases the confidence interval of the best algorithm overlaps with that of CHIM-NSGA. The values of Spread metric show that although the solutions obtained by the WASF-GA algorithm are distributed more evenly in most cases, their corresponding confidence intervals usually also overlap with those of CHIM-NSGA (Table 4).

Table 1. Test problems and reference points used in the evaluated algorithms

Problem	Number of objectives	Number of variables	Reference point
ZDT1	2	30	(0.80, 0.60)
ZDT2	2	30	(0.80, 0.80)
ZDT3	2	30	(0.30, 0.80)
ZDT4	2	10	(0.80, 0.60)
ZDT6	2	10	(0.78, 0.61)
DTLZ1	3	7	(0.20, 0.20, 0.20)
DTLZ2	3	12	(0.60, 0.70, 0.70)
DTLZ3	3	12	(0.60, 0.70, 0.70)
DTLZ4	3	12	(0.60, 0.70, 0.70)
DTLZ5	3	12	(0.60, 0.70, 0.80)
DTLZ6	3	12	(0.60, 0.70, 0.80)

Table 2. Mean of PR metric

Problem	CHIM-NSGA	WASF-GA
ZDT1	100.00	100.00
ZDT2	100.00	99.00
ZDT3	100.00	100.00
ZDT4	100.00	62.70
ZDT6	100.00	99.00
DTLZ1	100.00	99.67
DTLZ2	100.00	92.18
DTLZ3	90.00	92.13
DTLZ4	100.00	88.93
DTLZ5	100.00	91.50
DTLZ6	100.00	91.50

Table 3. Mean and confidence intervals of Generational Distance metric

Problem	CHIM-NSGA	WASF-GA	R-NSGA-II
ZDT1	**3.52E-04** ±8.46E-05	1.39E-03 ±9.95E-05	1.43E-03 ±8.71E-04
ZDT3	**4.87E-04** ±9.79E-05	1.66E-03 ±1.95E-04	1.29E-03 ±8.95E-04
ZDT3	**1.76E-04** ±7.44E-05	7.36E-04 ±6.33E-05	1.27E-03 ±6.76E-04
ZDT4	**3.03E-04** ±7.72E-05	2.74E-01 ±3.84E-02	1.01E-03 ±7.11E-04
ZDT6	3.20E-04 ±7.58E-05	1.35E-02 ±9.01E-04	**3.09E-04** ±5.91E-05
DTLZ1	**1.19E-03** ±1.43E-04	5.64E-03 ±6.08E-05	4.30E-03 ±1.79E-03
DTLZ2	**2.43E-03** ±4.20E-04	2.95E-03 ±9.31E-05	2.57E-03 ±4.16E-04
DTLZ3	5.42E-03 ±1.03E-03	**4.92E-03** ±6.54E-04	3.29E-02 ±6.65E-03
DTLZ4	**2.32E-03** ±2.16E-04	2.96E-03 ±9.22E-05	2.58E-03 ±4.12E-04
DTLZ5	**5.43E-04** ±1.23E-04	7.66E-04 ±8.34E-06	1.05E-03 ±8.18E-04
DTLZ6	**3.72E-04** ±0.00E+00	4.10E-03 ±0.00E+00	4.84E-04 ±0.00E+00

Table 4. Mean and confidence intervals of Spread metric

Problem	CHIM-NSGA	WASF-GA	R-NSGA-II
ZDT1	**5.86E-03** ±1.33E-03	9.11E-03 ±6.50E-04	8.67E-03 ±3.20E-03
ZDT2	5.19E-03 ±1.27E-03	**3.10E-03** ±9.70E-04	1.52E-02 ±7.79E-03
ZDT3	8.80E-03 ±1.02E-03	9.65E-03 ±1.71E-04	**6.79E-03** ±2.39E-03
ZDT4	4.17E-03 ±1.33E-03	**3.36E-03** ±7.27E-04	9.12E-03 ±4.26E-03
ZDT6	**7.69E-03** ±8.57E-04	8.68E-03 ±3.10E-03	1.38E-02 ±6.50E-03
DTLZ1	2.44E-03 ±4.54E-04	**2.09E-04** ±5.36E-05	7.78E-02 ±1.97E-02
DTLZ2	3.02E-03 ±6.08E-04	**5.80E-04** ±6.63E-05	9.86E-03 ±2.84E-03
DTLZ3	4.74E-03 ±1.01E-03	**5.18E-04** ±1.53E-04	2.43E-02 ±6.50E-03
DTLZ4	3.61E-03 ±7.25E-04	**6.21E-04** ±6.07E-05	1.16E-02 ±4.12E-03
DTLZ5	3.97E-03 ±7.79E-04	**5.94E-04** ±5.00E-05	2.13E-02 ±1.23E-02
DTLZ6	5.00E-03 ±0.00E+00	**9.14E-04** ±0.00E+00	9.86E-03 ±0.00E+00

Concluding, the experimental investigation shows that all the solutions provided by CHIM-NSGA lie into the ROI for all but one problem. WASF-GA shows worse results with regard to the PR metric: in most cases, there are solutions obtained by WASF-GA outside the ROI. According to the Generational Distance metric, the CHIM-NSGA algorithm approximates the Pareto front better than the other two investigated algorithms. It should be noted that the proposed CHIM-NSGA is able to cope with the problem of approximating the whole ROI accurately while maintaining the sufficiently good distribution of the obtained solutions using a relatively small number of generations, i.e. in reasonable computational time. This property is especially important when optimizing many objective problems.

As a rule, EMO algorithms are iterative and each iteration consists of several stages: evaluation of objective functions, Pareto dominance ranking, genetic operations, memory allocation and management, and other computations. The most computationally expensive part of such approaches is the dominance ranking operators. One of the most widely used procedures is the fast non-dominated sorting (FNDS) [9], which is used in the state-of-the-art EMO algorithm NSGA-II and its extensions. As proved in [13], FNDS consumes most of the computational burden of the EMO algorithms. The usage of HPC techniques allows us to speedup this procedure (while whole EMO algorithm) significantly. During the STSM visit our previously developed parallel versions of FNDS [14-15] were incorporated into the CHIM-NSGA algorithm.

During the STSM the works on the application of the CHIM-NSGA for solving the different scale multi-objective facility location problems has been started also. Moreover, the results archived during the STSM visit were presented and discussed with the members of the "High-Performance Computing – Algorithms" group.

Finally, during my STSM we have submitted a research paper called "A reference point-based evolutionary algorithm for approximating regions of interest in multiobjective problems" to the *Journal of Global Optimization* (IF2015: 1.219). It is also planned to present new results at the Optimization 2017, IFORS 2017 and to prepare a research paper where the application of the developed CHIM-NSGA algorithm to multi-objective facility location problems will be investigated.

This research of the STSM is enclosed into the Working Groups WG1: "WG Theory-Driven Applications" and WG2: "WG Practice-Driven Theory" of the COST Action ImAppNIO CA15140. A goal of the proposed STSM action corresponds to the ImAppNIO focus on improvement of the applicability of nature-inspired optimization methods.

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